Letters

Modes and Losses of a Four-Mirror Ring Resonator

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Abstract—**An investigation is reported on modes and losses of a square ring resonator that consists of the numerical solution of the appropriate integral equation for a number of values of the mirrors' aperture and spacing. It turns out that two sets of modes can exist: circulating modes and modes between two facing mirrors. The circulating modes are of the Fabry–Perot type, and their losses can be evaluated on the basis of the losses of suitably equivalent Fabry–Perot resonators.**

In a preceding paper [1] we reported some experimental results obtained on an X-band four-mirror ring resonator. Ring resonators have been applied as gyroscopes and sensitive interferometers [2] and more recently have been used as regenerative power amplifiers [3], [4].

This letter is concerned with a theoretical study of the modes and losses of a ring resonator constituted by four flat mirrors placed at the vertices of a square normal to the square diagonals (Fig. 1) [5].

Limiting the study to the infinite-strip case and with the usual approximations, the integral equation was written by taking as reference surface one of the four mirrors (M_1) and considering the direct contribution from the facing mirror as well as the reflections from the intermediate mirrors under the assumption of waves circulating only in one direction:

$$\sigma_m u_m(P_3) = \frac{\exp(i\pi/4)}{\lambda^{1/2}} \int_{M_1} u_m(P_1) K(\rho_{13})\, ds_1 + \frac{\exp(i\pi/2)}{\lambda^{1/2}} \cdot \int_{M_1}\int_{M_2} u_m(P_1) K(\rho_{12}) K(\rho_{23})\, ds_1\, ds_2 \quad (1)$$

$$K(\rho_{rs}) = \frac{\exp(ik\rho_{rs})}{(\rho_{rs})^{1/2}} \cos\theta_{rs}, \qquad k = 2\pi/\lambda$$

where

- ρ_{rs} distance between any two points on two mirrors;
- θ_{rs} angle between ρ_{rs} and the normal to the mirror;
- $u_m(P)$ mth eigenfunction or mode of the resonator;
- σ_m corresponding eigenvalue.

The complete round trip can be obtained by applying this equation twice.

The problem can be visualized by considering the resonator in the equivalent form of a traveling-wave system or beam waveguide. In this case, two systems coexist: one system (Fig. 2) consists of a sequence of tilted apertures with a periodicity equal to the spacing d, and the other is formed by a series of parallel apertures spaced by $d' = d/(\cos \pi/4)$. From this configuration one can predict the existence, confirmed by the numerical results, of two sets of modes: circulating modes and modes between facing mirrors.

A numerical iterative procedure was used to solve the integral

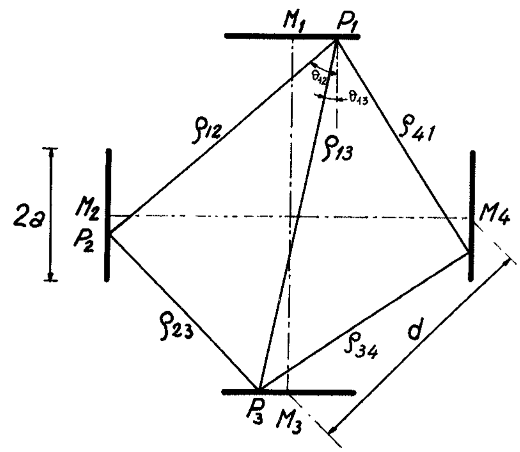

Fig. 1. Four-mirror ring resonator. Complete geometry of system.

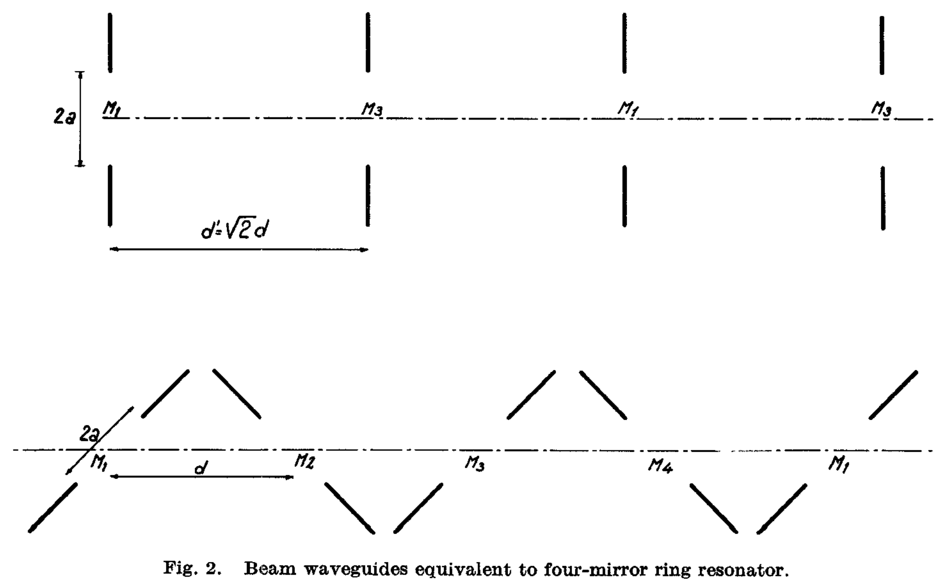

Fig. 2. Beam waveguides equivalent to four-mirror ring resonator.

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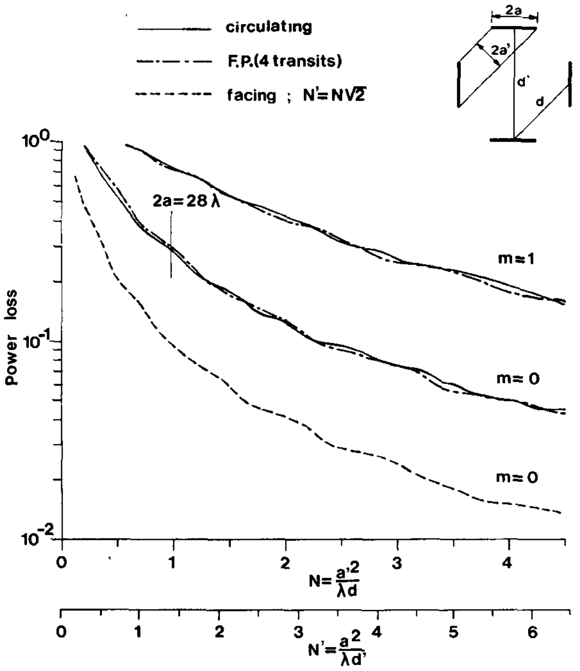

Fig. 3. Computed power loss per round trip for first even and odd modes versus $N = a'^2/\lambda d$, compared with the four-transit power loss of the equivalent Fabry–Perot. Dotted line refers to facing mode loss versus $N' = a^2/\lambda d'$.

equation for different resonator sizes, taking an input function with a constant phase-front normal to the line joining the centers of two consecutive mirrors.

The computations were limited to the first even and odd modes. The power losses per round trip for such circulating modes are shown in Fig. 3 (continuous lines) plotted versus Fresnel number $N = a'^2/\lambda d$ with $2a'$ being the actual aperture of the mirror projected in the beam direction ($2a' = 2a \cos \pi/4$). The dotted–dashed lines correspond to the losses per four transits of a Fabry–Perot having mirror aperture $2a'$ and spacing d equal to the distance between the centers of two adjacent mirrors.

The lower dashed curve represents the losses of the "facing zeroth-order mode" plotted versus Fresnel number $N' = a^2/\lambda d'$ with $d' = d/(\cos \pi/4)$. The Fresnel number N' is obtained from N, $N' = N\sqrt{2}$. Accordingly, the facing mode losses are always lower than those of the circulating mode. The presence of one or the other type of modes does not only depend on the assumed excitation, because the diffracted energy of the circulating mode will always excite the facing mode, and as the iteration process is continued, only the mode normal to the mirrors remains. Hence, below a given mirror size, the steady-state circulating solution can be obtained only by inhibiting the contribution of the facing mirror, which physically is equivalent to blocking the direct path through the center of the ring resonator. In our case this occurred for about $2a = 28\lambda$ ($2a' = 20\lambda$).

Fig. 4 shows the phase shifts of the modes plotted versus N and N', respectively.

The zeroth-order mode pattern has a configuration across the beam of the type of the zeroth-order mode of the planar Fabry–Perot resonator and a similar configuration across the mirror, but with a slight asymmetry depending on the assumed circulation sense. It is to be noted that when considering the first odd mode, due to the tilting of the mirrors with respect to the circulating mode wavefront, although using an odd input function, a mode conversion occurs that gives rise, after a certain number of iterations, to the even lowest order mode. Consequently, a method [6] has been applied which substantially consists of eliminating at each iteration the component of the zeroth mode by the use of the orthogonality of the solutions. Another anomalous behavior of such mode is that for each value of N, according to the chosen combination mirror aperture/spacing, it is possible to find a mode having the same loss and phase shift as the odd mode but a completely different field configuration.

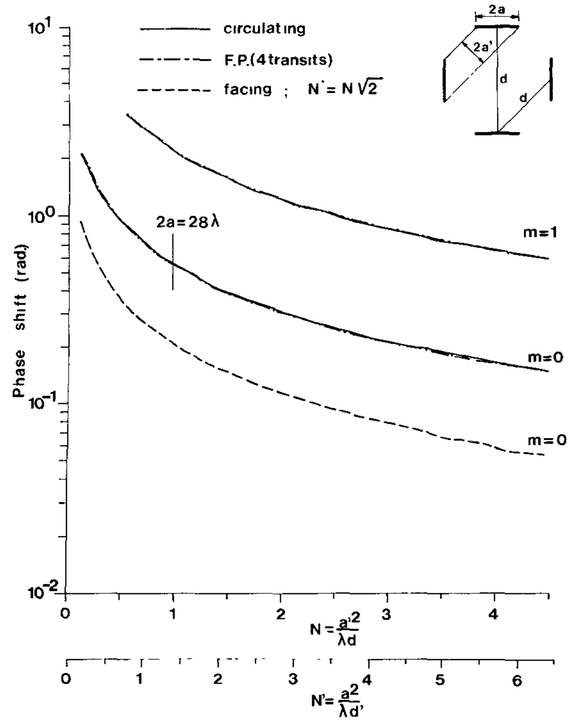

Fig. 4. Computed phase shift per round trip for first even and odd modes versus $N = a'^2/\lambda d$, compared with the four-transit phase shift of the equivalent Fabry–Perot. Dotted line refers to facing mode phase shift versus $N' = a^2/\lambda d'$.

In conclusion, these results show that in a four flat-mirror ring resonator two sets of modes exist: circulating and facing modes, both essentially of the planar Fabry–Perot type. Facing modes always have lower losses, and in some cases the diffracted energy is so high that only the facing mode is excited.

The losses of the circulating modes can be evaluated on the basis of the losses per four transits in the equivalent Fabry–Perot resonator.

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Dissipative Parameters in Ferrites and Insertion Losses in Stripline *Y* Circulators below Resonance

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***Abstract*—Extensive microwave loss measurements have been performed at the frequency of 1.3 GHz on below resonance stripline *Y* circulators loaded with aluminum doped YIG. External χ'' have been measured on the same compositions. Also, dielectric loss**

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